How a Mathematics lecturer teaches…

– transposition – rearranging – solving – changing the subject of –

equations/formulae

- We **avoid** using terms "move", "cancel", "bring across equals sign", "cross-multiply" as these lead to student errors such as:

$$60 + R = 0 \Rightarrow R = 60 \quad \times$$

$$\frac{60}{R} + 10 = 1 \Rightarrow R = 70 \quad \times$$

$$R = \frac{R + 3}{2R} + 1 \Rightarrow R = 3 \quad \times$$

- An **equation** says that a pair of quantities are equal. If we **do the same "thing"** (operation) **to both "sides"** of the equation we get another pair of equal quantities – another equation:

$$x = 16 \qquad x = 16 \qquad x = 16$$

$$\frac{x}{\mathbf{2}} = \frac{16}{\mathbf{2}} \qquad x + \mathbf{3} = 16 + \mathbf{3} \qquad \sqrt{x} = \sqrt{16}$$

- We **simplify** formulae/equations step-by-step removing (getting rid of) things that are in the way of our subject by applying **opposite/inverse operations**.

- It helps to use the peeling of an onion - removing outer layers before getting to the core - as an analogy of getting to the subject in an equation.

Want to		*do to* ***both*** *sides*
get rid of s^2	$A = \pi r^2 + s^2$	$- s^2$
get rid of π	$A - s^2 = \pi r^2$	$\div \pi$
get rid of the **square**	$\frac{A - s^2}{\pi} = r^2$	**square root**
	$\pm\sqrt{\frac{A - s^2}{\pi}} = r$	

Suggestions

how to quickly remind students about the main concepts

1. **Remind** the students that
   - we can do anything to an equation as long as we do the same thing to both sides;
   - we simplify/rearrange equations step by step by applying opposite/inverse operations.

Mutually opposite (inverse) operations
Addition and subtraction $x + \mathbf{5} = 3 \quad \big\vert \; \mathbf{-5}$ $x = -2$
Division and multiplication $\mathbf{2} \times p = 7 \quad \big\vert \; \mathbf{\div 2}$ $p = \frac{7}{\mathbf{2}}$
Square root and squaring both sides $\sqrt{x+3} = a + b \quad \big\vert \; \mathbf{\square^2}$ $x + 3 = (a+b)^{\mathbf{2}}$
Exponent and logarithm $\ln(x+3) = a + b \quad \big\vert \; \mathbf{e^{\square}}$ $x + 3 = e^{(a+b)}$

2. **Show** 2-3 examples from your field. Perhaps one relatively easy and a harder one.

Make $\boldsymbol{a}$ *the subject of*	do to **both** sides
$v = u + at$	$\mathbf{-u}$
$v - u = at$	$\mathbf{\div t}$
$\frac{v-u}{t} = a$	

Express $\boldsymbol{r}$ *from*	do to **both** sides
$p = \frac{r^2 + q^2}{L}$	$\mathbf{\times L}$
$pL = r^2 + q^2$	$\mathbf{-q^2}$
$pL - q^2 = r^2$	**square root**
$\sqrt{pL - q^2} = r$	

3. **Emphasise** that it is OK to do many small (but correct) steps. It is also OK to think only of the next small step instead of 'having to plan the entire route in detail from the start'.

4. **Give** the students the memento with tips on transposition to keep and consult when needed. The memento can be printed on A4 paper and folded in two to look like an oversized bookmark. There is a link in the memento to a webpage with worked out examples.